PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 10 MARCH 15, 1924 Number 3

DURATION OF MOLECULES IN UPPER QUANTUM STATES

By Richard C. Tolman

Norman Bridge Laboratory of Physics, California Institute of Technology

Communicated January 17, 1924

In an article which has been presented to the *Physical Review* for publication the author has shown the possibility of calculating from data on the intensity of absorption lines the mean life τ of atoms and molecules in excited quantum states, and the specific rate A_{21} at which they spontaneously leave the excited state 2 to return to the lower state 1. The calculations are based on Füchtbauer's[1] method of determining the probability that molecules will be activated by the absorption of a quantum of light, combined with Einstein's[2] views as to the mechanism of light absorption and emission. The following equation is derived connecting A_{21} with experimentally determinable quantities

$$A_{21} = \frac{8\pi\nu^2}{c^2 N_1} \frac{p_1}{p_2} \int_0^\infty \alpha d\nu$$

A_{21} is the chance per unit time that a molecule in quantum state 2 will spontaneously jump to quantum state 1 in the absence of exciting radiation, ν is the frequency of the light emitted in such a jump, and p_1 and p_2 are the *a priori* probabilities of states 1 and 2, while N_1 and α are data obtained in some particular absorption experiment made on the line accompanying the change from state 1 to state 2, N_1 being the number of molecules per unit volume in the lower quantum state and α the absorption coefficient for light which is producing the quantum jump under consideration. The integral $\int \alpha d\nu$ is to be taken across the whole effective width of the absorption line.

Using the above equation and data which could be obtained either directly or indirectly from the literature, values of A_{21} and τ were calculated which are repeated in table I.

The first column of table I gives the substance experimented on, the second column gives the quantum numbers designating the quantum jump

TABLE I

SUBSTANCE	LINE n,j,k→n′,j′,k′	λ	p_1/p_2	A_{21}	τ
Mercury	6,1,1→2,2,2	2537Å	1/3	9.68×10^{8}	1.03×10^{-7}
Sodium	3,1½,1→				
	3,2½,2	5890Å	1/3	3.25×10^{8}	3.08×10^{-7}
	3,1½,2	5896Å	1/3	6.21×10^{8}	1.61×10^{-7}
Caesium	6,2½,1→				
	7,2½,2	4555Å	1/3	3.56×10^{5}	2.80×10^{-6}
	7,2½,2	4593Å	1/3	1.16×10^{3}	8.64×10^{-7}
	8,2½,2	3877Å	1/3	2.37×10^{5}	4.21×10^{-6}
	9,2½,2	3612Å	1/3	1.58×10^{4}	6.33×10^{-5}
	9,1½,2	3617Å	1/3	7.98×10^{4}	1.26×10^{-5}
Iodine	——	5461Å	—	3.90×10^{2}	2.94×10^{-3}
	m,n→m′,n′				
HCl	12,0 → 13,1	3.239μ	12/13	57.7	0.0174
	8,0 → 9,1	3.299μ	8/9	5.69	0.176
	3,0 → 4,1	3.394μ	3/4	1.75	0.571
	2,0 → 3,1	3.415μ	2/3	1.21	0.829
	1,0 → 2,1	3.439μ	1/2	1.94	0.515
	1,0 → 0,1	3.489μ	∞	∞	0
			1	4.48	0.223
			0	0	∞
	2,0 → 1,1	3.516μ	2	5.93	0.169
	3,0 → 2,1	3.544μ	3/2	3.41	0.293
	8,0 → 7,1	3.699μ	8/7	6.38	0.157
	12,0 → 11,1	3.846μ	12/11	33.1	0.0303
HCl	23/2,0 → 25/2,1	3.239μ	12/13	31.3	0.0320
	15/2,0 → 17/2,1	3.299μ	8/9	3.80	0.263
	5/2,0 → 7/2,1	3.394μ	3/4	1.52	0.660
	3/2,0 → 5/2,1	3.415μ	2/3	1.10	0.910
	1/2,0 → 3/2,1	3.439μ	1/2	1.87	0.535
	3/2,0 → 1/2,1	3.489μ	2	4.77	0.210
	5/2,0 → 3/2,1	3.516μ	3/2	3.34	0.299
	7/2,0 → 5/2,1	3.544μ	4/3	2.71	0.369
	17/2,0 → 15/2,1	3.699μ	9/8	8.58	0.117
	25/2,0 → 23/2,1	3.846μ	13/12	57.5	0.0174
HF	6,0 → 7,1	2.395μ	6/7	20.5 × $1/p_{HF}$	0.0488×p_{HF}^3
	2,0 → 3,1	2.475μ	2/3	8.19 "	0.122 "
	1,0 → 2,1	2.499μ	1/2	4.81 "	0.208 "
	2,0 → 1,1	2.578μ	2	13.6 "	0.0738 "
	5,0 → 4,1	2.671μ	5/4	13.1 "	0.0764 "
HF	11/2,0 → 13/2,1	2.395μ	6/7	11.37 "	0.0880 "
	3/2,0 → 5/2,1	2.475μ	2/3	6.88 "	0.145 "
	1/2,0 → 3/2,1	2.499μ	1/2	4.46 "	0.224 "
	5/2,0 → 3/2,1	2.578μ	3/2	8.52 "	0.117 "
	11/2,0 → 9/2,1	2.671μ	6/5	17.94 "	0 0557 "

under consideration, (n, j, k being respectively total, inner and azimuthal quantum number, and m, n being respectively rotational and oscillational quantum number), the third column gives the ratio of *a priori* probabilities, the fourth column the values of A_{21} and the fifth column the mean life τ. This latter quantity is merely the reciprocal of A_{21} and is the mean life of molecules in quantum state 2 provided the molecules are leaving that state only by the one process of jumping to state 1. The values for the halides are calculated for the same lines both on the theory of whole and on the theory of half azimuthal quantum numbers.

The following conclusions are drawn as to the values obtained. They agree with the meagre data on mean life obtained by other experimental methods and hence lend support to Einstein's 1917 position as to the emission and absorption of radiant energy. Values of mean life may vary for different quantum states at least over the range 1 to 10^{-8} seconds. The rate of decay is not a simple function of the frequency of the emitted light. The rate corresponding to the emission of a line of high frequency may be greater or less than for a line of lower frequency. The data now available for the alkali doublets $1s - mp_1$ and $1s - mp_2$ indicate a higher rate of decay the smaller the change in total quantum number for the line under consideration. The rate of decay from a given mp_1 state is m times as great as from the corresponding mp_2 state, (already stated in another form by Füchtbauer and Hofmann). In the case of the rotation-oscillation spectra of the hydrogen halides, the rate of decay is greater for quantum states with one unit of oscillation and many of rotation than for those with one unit of oscillation and only a few of rotation; and in the case of different molecules but the same quantum numbers the rate of decay is greater for the molecule with the greater frequency of oscillation.

In the article mentioned the importance of determining further accurate values of $\int \alpha d\nu$ and developing other methods of determining rate of decay is emphasized. The possibility and method of calculating absolute values of A_{21} from the Bohr correspondence principle is also indicated. It is hoped to make such considerations the subject of a later paper.

[1] Füchtbauer, *Physik. Zs.*, **21**, 322 (1920).

[2] Einstein, *Ibid.*, **18**, 121 (1917).

[3] p_{HF} is the unknown partial pressure of undissociated hydrogen fluoride HF present in the absorption tube. The results, given in the table, have relative significance and since p_{HF} was almost certainly less than one, some absolute significance.

ON THERMAL EMISSION AND EVAPORATION FROM WATER

By M. Allen

Department of Physics, Mount Holyoke College

Read before the Academy November 16, 1922

A quantity which we may call the thermal emission from a water surface has been determined consisting of radiation, convection and conduction under the given conditions, comparable with the emissivity determined by Macfarlane[1] in 1871 for copper spheres, later verified and extended by Bottomley[2,3,4] who varied the pressure of the air in the enclosure and the temperature of the enclosure, and at low pressures was able to verify Stefan's fourth power law. But one other attempt seems to have been made, that of Karl Siegel[5] in his investigation of the heat emission of various stones; however, his work is based entirely upon a hypothetical law of Paschen's. In dealing with the problem of the thermal emission from water, the rate of evaporation has been found and studied as a function of the vapor pressures of water at the temperature of the hot water and of the surrounding envelope.

The experiment is quite tedious as it must run for two or more hours in order that equilibrium be first obtained and then maintained for at least half an hour before any readings are taken. The rate of evaporation is determined by timing the loss in weight. The quantity of energy given up per second by unit area of water surface, per unit difference of temperature between the surface and surrounding bodies is the quantity which has been measured in this work; it is expressed in gram calories per second per square centimeter per degree Centigrade. This has been called by Preston[6] and Poynting and Thomson[7] the emissivity of a surface. The energy loss in this case involved convection and conduction as well as radiation losses. The standard environment employed is a dry black surface, but with this apparatus the black inner surface of the cone which performs this function is of necessity covered with a thin layer of the condensed water and is wet. In this experiment, the electrical input is expended in two ways,—in evaporating the water and in emission from the water surface. Knowing the rate of evaporation and using the values of the latent heat of evaporization of water, the amount of heat required per second for the evaporation is determined. From the remainder of the input, the thermal emission is then computed.

There are several sources of error evident. Absolutely steady conditions of temperature and evaporation are perhaps never fully obtained although closely approximated. At the lower temperatures the evaporation rate can be duplicated very closely from day to day. At the higher temperatures, under apparently identical conditions, this rate varies

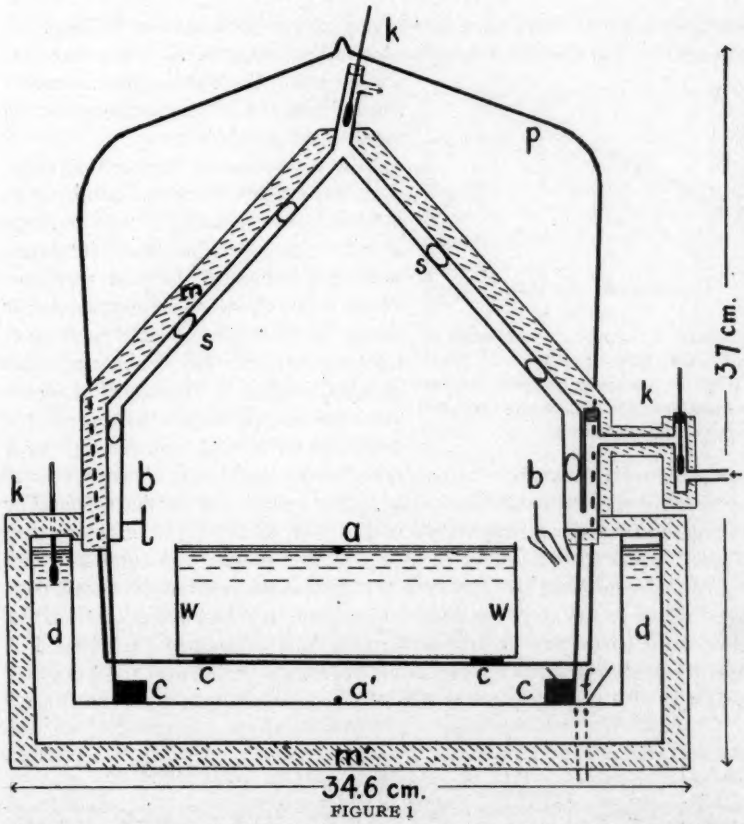


FIGURE 1

The copper dish *ww*, nickel-plated, holds the water which is heated electrically by a coil enclosed in small glass tubing. This dish is set on the balance pan *p* which is adjusted so that it is perfectly free to move and so allows the loss of weight by evaporation to be timed. The outside water jacket *dd*, of copper nickel-plated and also electrically heated, is kept at the same temperature as the inner dish, which prevents loss of heat from the inner dish other than from the water surface. The water-cooled cone *ms* is kept at 20.5° C thus maintaining a constant temperature difference between the water surface and the environment to which it is losing heat. *Ss* show the cross sections of a spiral rubber wall to direct the flow of the water which has been previously carefully regulated both as to pressure and as to temperature so that the temperature of the cone is kept very constant. The temperatures involved are given by the thermometers *kkk* and the thermocouples of copper-ideal wire *aa'*. The thermocouple *a* is floated near the surface of the water, being supported by a small piece of cork and protected by a flexible casing of small rubber tubing. The water which condenses on the black inner surface of the cone is collected in the inclined gutter *b* and flows out the funnel *f*. Heat losses are prevented as much as possible: both water jacket and cone are enclosed in felt *mm'*; cork insulators *cccc* are used between bodies at different temperatures; the inner dish is never allowed to touch the bottom of the water jacket; and the cone is carefully adjusted so that its bottom is on a level with the top of the inner dish.

sometimes 2% or more from day to day and even from hour to hour. A change of 1% in the rate of evaporation means, other things being constant, a variation at the higher temperatures of two or three times that amount in the value of the emissivity.

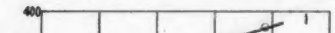


FIGURE II

Curve 1. Emissivity of water in cal./cm.² deg. sec.; Curve 2. Emissivity of blackened copper—Macfarlane; Curve 3. Emissivity of polished copper—Macfarlane.

Determinations of the thermal emission for eight different temperature differences were made, covering a range of 40° C. at approximately 5° intervals. The temperature of the cone was kept about 20.5° C. The resulting curve is shown in figure II compared with Macfarlane's curves for blackened and polished copper. The thermal emission for water could be represented within experimental error in the given range by straight line, but the parabolic line fits better and is moreover in agreement with the results for copper. The equation of the line as found by the method of least squares is $10^6 e = 215 + 4.87\,(t - t_0) - .022\,(t - t_0)^2$. Table I gives the experimental values of the emissivity as compared with the values computed by this formula. The experimental points are averages of two to five days' readings which in no case had a maximum deviation of the single day's readings from the final average of more than 1%. It will be noted that the agreement between the experimental and computed points is with one exception within 1%.

TABLE I

TEMPERATURE DIFFERENCE	EMISSIVITY (EXPERIMENTAL)	EMISSIVITY (COMPUTED)	PER CENT DIFFERENCE
6.40°	.000245	.000245	0.0%
10.05°	.000262	.000262	0.0%
15.30°	.000286	.000284	0.7%
19.90°	.000304	.000303	0.3%
25.50°	.000321	.000325	1.2%
30.00°	.000342	.000341	0.3%
34.80°	.000356	.000358	0.7%
38.75°	.000373	.000371	0.7%

TABLE II

(V−v) INCHES MERCURY	EVAPORATION		
	INCHES/HOUR (EXPERIMENTAL)	INCHES/HOUR (COMPUTED)	INCHES/HOUR (FITZGERALD)
.328	.00333	.00438	.00462
.570	.00680	.00774	.00837
1.00	.0135	.0140	.0152
1.50	.0214	.0218	.0228
2.24	.0354	.0344	.0374
3.04	.0487	.0490	.0546
3.96	.0705	.0677	.0743
5.02	.0900	.0912	.1006

A second result is the evaporation curve, figure III, in which the rate of evaporation expressed in the rate of lowering of the liquid surface in inches per hour is plotted against the difference in inches of mercury pressure between the vapor pressure of the water V and the vapor pressure of water at the temperature of the cone v. The rate of evaporation of water is a very complex quantity depending on many different factors,

the most important of which are the difference in vapor pressure between the water and the surroundings, the area of the evaporating surface, its distance below the top of the dish—the so-called rim effect—and the velocity of air currents above the surface. The conditions of this experiment are those of no air currents and of no variation in area. The rim effect has been neglected in plotting this curve; if it exists, it is very small. Using the method of least squares, the evaporation curve is given by the equation $10^3\ E = 13\ (V - v) + 1.03\ (V - v)^2$. Except for very small values of $(V - v)$ it represents the experimental results with fair accuracy. The equation is expressed in English units, inches per hour and inches of mercury pressure, that it may be the more easily compared with the curve obtained in 1886 by FitzGerald[8] giving the relation as he found it for the evaporation of water in still air. His experiments were carried out on a larger scale, with dishes 14.85 inches in diameter uncovered in a laboratory room. His equation in the same units is $10^3\ E = 14\ (V - v) + 1.2\ (V - v)^2$. The two curves agree in shape and vary only by 10% of the new values. When one considers how widely scattered evaporation curves usually are and how entirely different the two methods of determination are, the agreement is quite satisfactory. We should expect his curve to lie below rather than above the new one, since it has been considered as proved[9] that the rate of lowering of a water level by evaporation is smaller for large surfaces than for small; there are, however, many conflicting tendencies involved and it is an interesting fact that the rate of free evaporation has been so nearly checked by an experiment on forced evaporation.

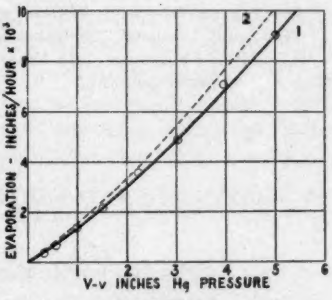


FIGURE III

Curve I. Result of this research; Curve II. Desmond FitzGerald's evaporation curve.

I am under deep obligations to my late teacher Prof. A. G. Webster for suggesting the subject of this investigation and for presenting the conclusions to the NATIONAL ACADEMY OF SCIENCES and to Prof. C. L. Norton for supervision of the experimental details in the Rogers' Laboratory of Physics at the Massachusetts Institute of Technology.

[1] Macfarlane, Donald, "Experiments Made to Determine Surface-Conductivity for Heat in Absolute Measure." *Proc. Roy. Soc.* **20,** p. 90, 1871.

[2] Bottomley, J. T., "On Thermal Radiation in Absolute Measure." *Phil. Trans. Roy. Soc.*, **A,** p. 591, 1893.

[3] Bottomley, J. T., "Thermal Radiation at Very Low Temperatures." *Brit. Assoc. Rpt.*, p. 330; 1905.

[4] Bottomley, J. T. and King, F. A., "Thermal Radiation in Absolute Measure at Very Low Temperatures." *Phil. Trans. Roy. Soc.*, **208A,** p. 349, 1908.

[5] Siegel, Karl, "Uber das Emissionsvermogen von Gesteinen, Wasser und Eis." *Sitzungber. Akad. Wien*, **116, 2A**, p. 1203, 1907.

[6] Preston, T., *Theory of Heat*, p. 443.

[7] Poynting and Thomson, *Textbook of Physics, Heat.*, p. 94.

[8] FitzGerald, Desmond, "Evaporation." *Am. Soc. Civ. Eng. Trans.*, **15**, p. 581, 1886.

[9] Thomas and Ferguson, "On Evaporation from a Circular Water Surface." *Phil. Mag.*(*Ser. 6*), **34**, p. 308, 1917.

ON SECONDARY AND TERTIARY X-RAYS FROM GERMANIUM, ETC.

BY GEORGE L. CLARK[1] AND WILLIAM DUANE

JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY

In three previous notes we[2] have described experiments in which an X-ray beam, emergent from a secondary radiator at right angles to the primary beam from a tungsten target, was analyzed by means of a calcite crystal and an accurate ionization spectrometer with sensitive electrometer.

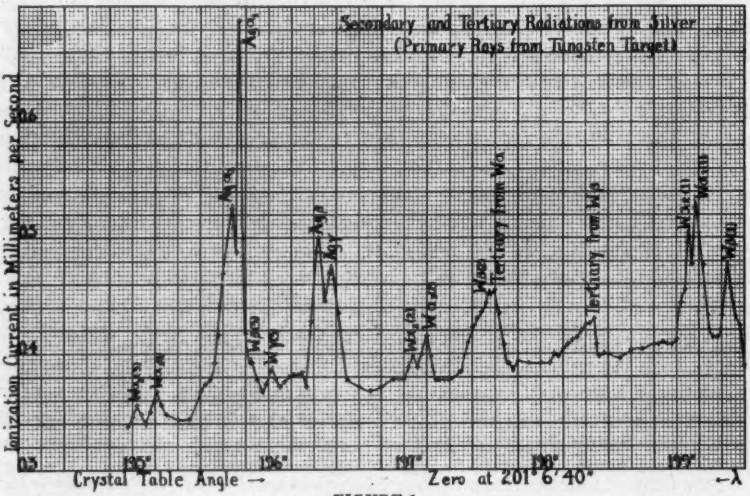


FIGURE 1

Results were given for 14 different chemical elements from carbon (atomic number 6) to neodymium (atomic number 60). We were able to identify and to measure the wave-lengths of three kinds of X-radiation in the secondary beam: (1) scattered rays with the same wave-lengths as the tungsten K series lines in the primary beam; there was no evidence

of a softening of these rays as demanded by A. H. Compton's interesting theory of the scattering of radiation quanta by single electrons with recoil; (2) fluorescent secondary rays, characteristic of the element in the radiator; and (3) tertiary rays produced by the impact of secondary photo-electrons on atoms in the radiator. We showed that the short wave-length limit, λ, of the tertiary radiation (which appears as a broad band) is given by the expression $\lambda = \lambda_1\lambda_2/(\lambda_2-\lambda_1)$, where λ_1 denotes a primary wave-length and λ_2, is a critical absorption wave-length of the chemical element in the radiator. The shift from the wave-length of the unmodified ray in the scattered beam is $\Delta\lambda = \lambda_1^2/(\lambda_2-\lambda_1)$.

In this note we present additional experimental evidence bearing upon the phenomena just outlined.

1. *Secondary and Tertiary Radiation from Silver.*—The last of the three notes referred to (l. c.) contains the data for this experiment. The corresponding graph is now presented in Fig. 1 without further comment, since the tungsten peaks, the fluorescent silver peaks and the tertiary "humps" due to WKα and WKβ are all labelled on the diagram.

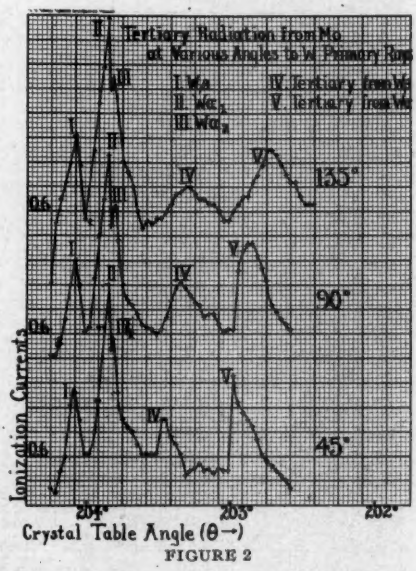


FIGURE 2

2. *The Effect of the Angle between Primary and Scattered Beams on the Tertiary Rays from Molybdenum.*—According to the theory, as represented by the equation $\lambda = \lambda_1\lambda_2/(\lambda_2-\lambda_1)$, the short wave-length limit and the angle of its reflection by a calcite crystal should be independent of the angle between the scattered beam which is being analyzed and the primary beam striking the radiator. On the other hand the shape of the tertiary humps and the angular positions of their *maxima* might be expected to vary, for a variety of reasons, such as filtration.

In order to test the effect of changing the angle, three experiments with a fixed molybdenum radiator were made under conditions identical except that the tungsten target tube was adjusted successively in such a way that the scattered beams passing through the spectrometer slits and reflected by the crystal made angles of approximately 135°, 90° and 45° with the primary beam. The results appear in Fig. 2. The three curves show

the peaks due to unmodified Wβ and Wα rays at the same crystal table angles, and also tertiary humps due to Wβ and Wα. These differ considerably from each other in appearance. For the 45° experiment the humps are fairly narrow and quite sharp on the short wave-length side. At 90° they are somewhat more diffuse, and at 135° they appear as practically symmetrical, broad peaks. The tops of the tertiary humps clearly occur at quite different angles as shown by the following readings: (45°) 202° 55′; (90°) 202° 51′; (135°) 202° 45′. An important characteristic, however, is that in all three cases the corresponding tertiary humps have the *same short wave-length limit*, i. e., they start at the same angles, 30′ and 1°, from the beginning of the Wα peak for the tertiary radiations due to Wβ and Wα respectively. This experiment, therefore, contributes strong evidence in support of the existence of tertiary radiation and of the validity of the simple equation expressing its short wave-length limit. The shifts in the humps with the angle of incidence of the primary rays parallels the results of A. H. Compton's experiments with molybdenum primary rays and a carbon radiator.

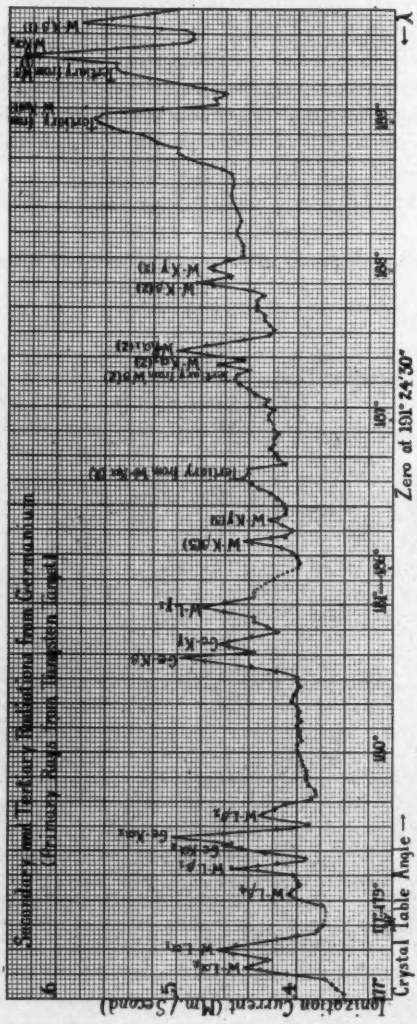

Secondary and Tertiary Radiations from Germanium
(Primary Rays from Tungsten Target)
Ionization Current (Mm./5 second)
Crystal Table Angle
Zero at 191° 24′ 30″

FIGURE 3

On a photographic record of the secondary radiation, with the proper time of exposure, the tops only of these tertiary humps would show. It would take a *very* much longer exposure to bring out the short wave-length limits.

3. Secondary and Tertiary Radiation from Germanium.—An experiment with metallic germanium as secondary radiator furnished some exceedingly interesting results. Professor L. M. Dennis of Cornell University kindly sent us a sample of the pure metal in the form of a quarter section of a large button-shaped melt with one edge highly polished. Considering that our previous work has been done with polished, fairly thin plates having comparatively large areas, strong radiation from this sample was not anticipated. However careful adjustment of the polished face (which had an area of only 2.5 cm.^{2}) so that the scattered beam grazing its surface might be analyzed, produced surprisingly large ionization currents. Fig. 3 and the following table I contain the results.

TABLE I

THE WAVE-LENGTHS OF SECONDARY AND TERTIARY RAYS FROM GERMANIUM

SPECTROMETER ANGLE	GLANCING ANGLE, θ	WAVE-LENGTH, λ	RADIATION	ORDER	λ KNOWN
191°-24′-30″					
189°-39′-20″	1°-45′-10″	0.1852	W-Kβ	1	0.1842
189°-25′-45″	1°-58′-45″	0.2091	W-Kα	1	0.2086
189°-22′- 0″			Top of tertiary due to W-Kβ	1	
189°- 0′- 0″	2°-24′-30″	0.2545	Tertiary due to W-Kα	1	0.2566 (calc.)
188°- 1′- 0″	3°-23′-30″	0.1791	W-Kγ	2	0.1790
187°-55′- 0″	3°-29′-30″	0.1844	W-Kβ	2	0.1842
187°-27′-30″	3°-57′- 0″	0.2086	W-Kα_1	2	0.2086
187°-22′- 0″	4°- 2′-30″	0.2134	W-Kα_2	2	0.2134
187°-20′- 0″			Limit tertiary due to W-Kβ	2	
186°-42′- 0″			Limit tertiary due to W-Kα	2	
186°-19′-10″	5°- 5′-20″	0.1791	W-Kγ	3	0.1790
186°-10′-20″	5°-14′-10″	0.1842	W-Kβ	3	0.1842
180°-59′- 0″	10°-25′-30″	1.0958	W-Lγ_1	1	1.0955
180°-44′- 0″	10°-40′-30″	1.1228	Ge-Kγ	1	1.121
180°-38′-30″	10°-46′- 0″	1.1314	Ge-Kβ	1	1.131
179°-34′-50″	11°-49′-40″	1.2413	W-Lβ_2	1	1.2417
179°-25′- 5″	11°-59′-25″	1.2581	Ge-Kα_1	1	1.257
179°-22′- 0″	12°- 2′-30″	1.2634	Ge-Kα_2	1	1.261
179°-13′- 0″	12°-11′-30″	1.2789	W-Lβ_1	1	1.2792
179°- 2′-30″	12°-22′-30″	1.2968	W-Lβ_4	1	1.2987
177°-20′- 0″	14°- 4′-30″	1.4728	W-Lα_1	1	1.4735
177°-13′- 0″	14°-11′-30″	1.4857	W-Lα_2	1	1.4845

The spectrum curve leads to the identification of the following kinds of radiation: (1) scattered K series tungsten rays with primary wave-lengths, through three orders: (2) scattered L series tungsten rays with unmodified wave-lengths (the intensities are surprisingly great. This portion of the spectrum was determined at a different time from the K

series portion, under possibly more sensitive conditions); (3) fluorescent K series germanium rays, whose wave-lengths have here been measured more accurately than ever before; (4) tertiary rays due to W-Kβ and W-Kα *in two orders* (the first order tertiary hump due to W-Kβ is not separated from the W-Kα peak. The limit should come, by calculation, slightly less than 7′ from the beginning of the W-Kα peak); and (5) evidences of other tertiary radiation, as shown, for example, by the relatively great height of the W-Lα_2 peak. Calculation shows that tertiary rays produced by germanium L photo-electrons, which are removed from their orbits by impact of the W-Lβ rays, should appear at the angle of the W-Lα_2 peak. It appears, therefore, that tertiary rays from a multitude of sources and in intensities measurable by a sufficiently accurate and sensitive apparatus are present in the X-ray beams scattered by matter.

4. Scattering from a Paraffin Block.—In our previous experiments with carbon radiators and tungsten primary rays we used a polished graphite plate. The W-Kβ and W-Kα rays scattered by the graphite had wave-lengths identical with those of the primary rays, and there was no comparable radiation at an angle 14′ larger, as demanded by A. H. Compton's theory. We have repeated these experiments, this time using a paraffin block as radiator. The results are identical with those obtained with graphite, a diagram for which is shown in Fig. 1 of the second note referred to above (l. c.). This answers the possible objection that the failure to obtain the shifted rays might be due to the crystalline nature of the graphite as opposed to the amorphous state of paraffin.

[1] NATIONAL RESEARCH FELLOW.

[2] These PROCEEDINGS, **9**, 413, 419 (Dec. 1923); **10**, 41, 47 (Jan. 1924).

EXHIBIT OF TELEPHONIC EXCITATION OF ACOUSTIC PRESSURE[1]

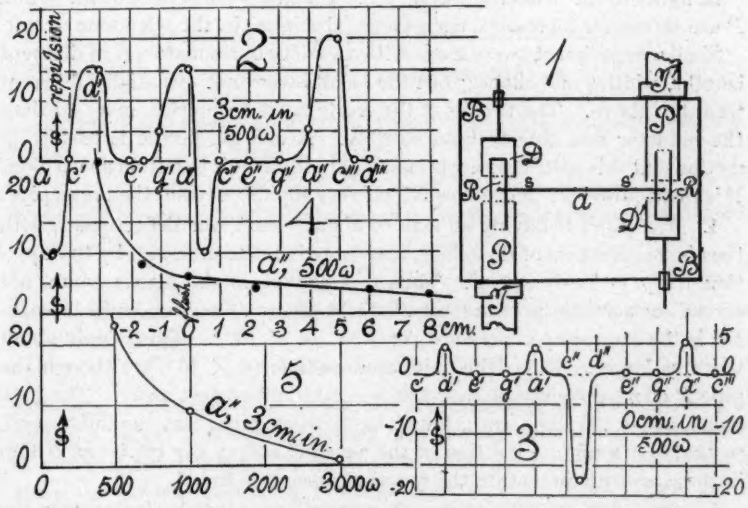
BY CARL BARUS

DEPARTMENT OF PHYSICS, BROWN UNIVERSITY

Communicated January 29, 1924

1. In *Science* (**24**, p. 155, 1921), I described a series of simple experiments bearing on the nature of the acoustic forces observed in connection with telephone-blown pipes. Having occasion to test this work recently, I obtained a series of repulsions as well as the former attractions. This induced me to repeat the experiments with the bifilar suspension shown in figure 1. Here TT' are the telephones excited by a small inductor and

break of variable frequency, each carrying the identical pipes PP' about 13 cm. long and 5 cm. in diameter with a note ranging from below b' to $\# c''$, depending on the degree to which the mouth is open. RR' are two identical cylindrical resonators of heavily varnished paper, about 9 cm. long and 2.2 cm. in diameter responding to a'' (in place), attached at right angles to a straw shaft ss', 26 cm. long. The deflections were observed with mirror and scale at a distance of 142 cm. To guard against air currents, the box BB' surrounds the whole, in such a way that the telephone pipes PP' may be more or less withdrawn, or that R may dip more or less into P. The period of the needle was about 30 sec., and the total weight 4.5 grams. The addition of adjustable closed cylinders DD' improves the damping, etc.



2. When the resonators R extend to about 3 cm. within P, the full telephonic current throws the light spot off scale. The current was therefore diminished by inserting 500 ohms resistance which reduced the sound to the loudness of an ordinary organ pipe. The survey in pitch (a to c''') then gave the deflections s reproduced in the graph figure 2, repulsion laid off positively. There are two marked attractions at b' the resonance pitch of P and at b an octave lower, none at b'' an octave higher. Hence the overtones of the telephone note are probably responsible for b, though its intensity is surprisingly large. Similarly there are 3 repulsions, the strongest at a'', the resonance pitch of R, while a' and d' are probably again referable to the successive overtones of the telephone note.

When the pipes P are partially withdrawn, so that the mouths of R and

P are coplanar, the deflections obtained were such as shown in figure 3. The repulsions (a'' a' d') have nearly vanished, but the attractions (now at the resonance pitch $\sharp c''$ of P) is still strong.

At a distance of 4 cm. between the mouths (organ-pipe loudness) the attractions at $\sharp c''$ also practically vanished; at a distance of 8 cm. quite so. One notes that P attractions are more persistent.

3. Thus the acoustic pressures fall off very rapidly with the distance apart of the systems R and P. In figure 4 with 500 ohms in circuit and the steady note a'', this relation is indicated, the mouth of the resonator R being within P for negative abscissas, in cms. The note is a little louder than in figure 3. The effect increases as the mouth of R approaches the telephone plate, though it is still 9 cm. off.

In figure 5, the deflections at a'' pitch with the mouth of R 3 cm. within P are shown for increasing resistances (abscissas) in the telephone circuit.

Similar experiments were made with a variety of resonators R of different lengths, widths, and shapes; but the results were not essentially different from the above. The tuning at the crests must be precise, more so than the inductor was able to hold steadily. Hence the needle is usually in motion and this with the long period makes it difficult to obtain sharp data. It is clear, however, that forces fall off very rapidly beyond the organ pipe.

4. The pipes R having an area of about 4 cm^2., the deflections s, with the above constants of the bifilar, may be reduced to pressures by the equation $10^4\ p = 7s$ dyne/cm^2. Thus the maxima in the figures would not exceed an acoustic pressure of $.0007 \times 20 = .014$ dyne/cm^2. If $p = 10^6$ is the atmospheric pressure $\Delta p/p = 1.4 \times 10^{-8}$. This is only about twice as large as Rayleigh's minimum audible (6×10^{-9}), although the pipe P could certainly be heard at several hundred feet away. The high value $\Delta p = .25$ dyne/cm^2., found for howling pipes was similarly small, so that but a minute fraction of the acoustic energy per cm.^{3} is locked up in these acoustic pressures, the remainder being radiated.

5. One may describe these phenomena[2] as a whole, by puttng the locally available energy in the form $p + \rho v^2/2 = p_0 + \Delta p_0$ where p_0 is the atmopheric pressure and Δp_0 an increment contributed by the acoustic vibration, increasing with its intensity, but constant within the region bounded by the hollow resonators. When the pipe P is in resonance, the waves are merely reflected, without nodes, within R. When P sounds loud enough, pressure p at the mouth of P may with increasing v have fallen below that of the atmosphere, while at the same time it has increased above atmospheric pressure at the closed or nodal end of P.[3] Thus the reduced pressure at the mouth of R is harbored within it, and it is thus thrust inward by the full atmospheric pressure on the outside. Conversely when R is in resonance, P merely reflects (without nodes) the waves emerging from R. If the vibration is of sufficient intensity, the pressure at the

mouth of R falls below that of the atmosphere, while the pressure within R, at its closed ($v = 0$, nodal) end rises above atmospheric pressure, so that R is now thrust outward from P.

[1] Advance note from a report to the Carnegie Inst., of Washington, D. C.

[2] "The Vigorous Treatment of Pulsating Sources" is given by V. Bjerknes, *Hydrodyn. Fernkr.*, J. A. Barth, 1902, Leipzig.

[3] This may be tested by lowering a disc within P. With this disc a neutral layer may be found about 2 cm. within P and a symmetrical neutral layer about 2 cm. from the telephone plate.

NEW TOPOLOGICAL INVARIANTS EXPRESSIBLE AS TENSORS

BY JAMES W. ALEXANDER

DEPARTMENT OF MATHEMATICS, PRINCETON UNIVERSITY

Communicated January 17, 1924

Let the Betti numbers of an arbitrary closed 3-dimensional manifold M_3 be

$$(P_1-1) = (P_2-1) = p$$

There will then be a fundamental set of p linearly independent non bounding closed surfaces

$$x_1, x_2, \ldots, x_p,$$

such that every non bounding closed surface s is related to the surfaces x_i by an homology

$$s \sim \alpha^i x_i$$

with integer coefficients α^i. (We are here following the convention now generally adopted in tensor analysis of assuming summation with respect to every index which occurs both as a subscript and as a superscript). Moreover, as was shown by Professor Veblen, there is an unique set of linearly independent non bounding closed curves

$$x^1, x^2, \ldots, x^p,$$

such that the algebraic sum of the points of intersection of a curve x^i with a surface x_j, with proper conventions as to sign, is equal to one or zero according as i is equal to or different from j. Thus, the intersection of a curve x^i with a surface x_j is the coefficient of $x^i x_j$ in the bilinear form

$$x^i x_i,$$

(summed for i, of course). In place of the fundamental surfaces x_i, we might have chosen a different set x_j

$$\bar{x}_j \sim \beta^i_j x_i$$

$$(\beta^i_j \text{ integers}, \| \beta^i_j \| = \pm 1)$$

The set of curves $\bar{x}^j$ dual to the set $\bar{x}_j$ would then have been related to the set x^i by homologies so chosen as to leave the form $x^i x_i$ invariant; that is to say, by homologies contragredient to the previous ones. It will therefore be understood in this paper that whenever we change the fundamental surfaces x_i we simultaneously make the contragredient change upon the fundamental curves x^i.

If the manifold M_3 have torsion, there will be certain additional curves y^i not expressible by homologies in terms of the x^i's and such that a suitable multiple of each curve y^i is homologous to zero, though not the curve itself. Even in this case, however, it will be possible to express an arbitrary curve c of the manifold in the form

$$c \sim \gamma_i x^i \qquad (\text{modd } y^i).$$

Now, consider a number σ_{ijk} which denotes the algebraic sum of the points of intersection of three fundamental surfaces x_i, x_j, x_k, with due regard to the sign of a point of intersection. From the relations

$$\sigma_{ijk} = -\sigma_{jik}, \text{ etc.},$$

$$\sigma_{(p+q)jk} = \sigma_{pjk} + \sigma_{qjk}, \text{ etc.},$$

we conclude at once that if we transform the fundamental system of surfaces x_i, the numbers σ_{ijk} are transformed like the components of a skew-symmetric tensor of the third order. We have therefore determined a topological invariant of the manifold expressible as a tensor.

A second invariant may be obtained as follows. Let $x_i . x_j$ be the curve of intersection of two fundamental surfaces x_i and x_j. Expressing the curve $x_i . x_j$ in terms of the fundamental curves x^i, we have

$$x_i . x_j \sim \rho_{ijk} x^k \qquad (\text{modd } y^i).$$

There is no difficulty in showing that the numbers ρ_{ijk} are also the components of a covariant tensor of the third order. The invariant ρ_{ijk} is essentially the one considered in my last note to these Proceedings. There, however, I did not take full advantage of the duality relation between the non bounding curves and non bounding surfaces of the manifold.

If the manifold M_3 have torsion, there are still other invariants of a similar sort to be found. I shall confine myself in this brief summary to the case where there are k coefficients of torsion, all of the same value τ. There must then be k independent curves

$$y^1, y^2, \quad , y^k,$$

not included among the linear combinations of the curves x^i, such that each curve y^i, when counted τ times is the boundary of an open surface z_i

$$z_i \equiv \tau y^i \sim 0.$$

Dual to the curves y^i are k linear combinations

$$y_1, y_2, \quad , y_k$$

of the surfaces z_i with the property that a curve y^i intersects a surface y_j once or zero times, modulo τ, according as i is equal to or different from j. In other words, the intersection of y^i with y_j is determined by the coefficient of $y^i y_j$ in the bilinear form

$$y^i y_j \qquad (\text{mod } \tau)$$

Let π^{ij} denote the number of times, modulo τ, that a curve y^i cuts the surface z_j bounded by y^j. The numbers π^{ij} are the components of a contravariant tensor of the second order, this time with respect to the variables y_i. In the general case, there will be a tensor π^{ij} corresponding to each value of the coefficients of torsion.

The tensors σ_{ijk} and ρ_{ijk} are not derivable from the Betti numbers and coefficients of torsion of a manifold, as may be seen by comparing the two manifolds given in my previous note. By means of the tensors π^{ij}, it is even possible to distinguish between manifolds with the same group. There are, for instance, two distinct manifolds of group 5 such that the Heegaard diagram of each may be formed on an anchor ring, but such that the characteristic curve of one diagram is ab^5, of the other a^2b^5. For the first manifold, the possible values of the tensor π^{11} are 1 and 4, modulo 5, depending upon the choice of the fundamental surface y_{11}; for the second manifold, the possible values are 2 and 3.

Evidently, the above discussion may be extended from three to n dimensions.

ON CERTAIN NEW TOPOLOGICAL INVARIANTS OF A MANIFOLD

BY JAMES W. ALEXANDER

DEPARTMENT OF MATHEMATICS, PRINCETON UNIVERSITY

Communicated January 14, 1924

In an n-dimensional manifold M_n let

$$(C_k^i) \qquad C_k^1, C_k^2, \ldots, C_k^p$$
$$(C_l^i) \qquad C_l^1, C_l^2, \ldots, C_l^q$$
$$(C_m^i) \qquad C_m^1, C_m^1, \ldots, C_m^r$$

be complete sets of linearly independent non-bounding circuits of dimensionalities k, l, and m respectively, such that

$$k + l > n,\ m = k + l - n.$$

Then, any k-dimensional circuit C_k is homologous to a linear combination of the circuits (C_k^i)

$$C_k \sim \alpha_1 C_k^1 + \alpha_2 C_k^2 + \ldots . \alpha_p C_k^p,$$

where the numbers $\alpha_1, \alpha_2, \ldots . \alpha_p$ are integers. Similarly, every l-circuit C_l is homologous to a linear combination of the circuits (C_l^i), and every m-circuit C_m to a linear combination of the circuits (C_m^i). Moreover, two circuits C_k^i and C_l^j intersect in a circuit C_m, so that we may write

$$C_m = C_k^i . C_l^j \sim \alpha_1 C_m^1 + \alpha_2 C_m^2 + \alpha_p C_m^p.$$

Construct a rectangular matrix J_{pq} of p rows and q columns, one row for each k-circuit of (C_k^i), one column for each l-circuit of (C_l^i), and let the element in the i-th row and the j-th column be the linear combination of the m-circuits (C_m^i) homologous to the intersection of C_k^i with C_l^j. The matrix J_{pq} may be modified by making linear transformations with integer coefficients and determinants ± 1 on the rows of the matrix, on the columns of the matrix, and on the symbols C_m^i occurring in the elements of the matrix. It is easy to prove that the group of all matrices so obtained by these linear transformations is a topological invariant of the manifold M_n, or, in other words, that if M_n and M_n^1 be homeomorphic manifolds, it must be possible to transform the matrix J_{pq} of M_n into the corresponding matrix J_{pq}^1 of M_n^1 by elementary transformations of the type just described.

The matrices J_{pq} are not dependent on the Betti numbers and coefficients of torsion of Poincaré, as a comparison of the following two examples will show:

First Example.—Consider the 3-dimensional manifold obtained by removing from spherical 3-space the interiors of six spheres, and deforming the residual space so as to match the surfaces of the spheres together pointwise in pairs, in such a manner as to obtain an orientable closed manifold M_n. The Betti numbers of the manifold are obviously

$$(P_1 - 1) = (P_2 - 1) = 3,$$

and there is no torsion. Moreover, the matrix J_{22} of intersections of the 2-circuits with one another is

(J_{22}):

	S_1^1	S_1^2	S_1^3
S_2^1	0	0	0
S_2^2	0	0	0
S_2^3	0	0	0

as may be seen by taking each pair of matched spheres as a fundamental 2-circuit.

Second Example.—Consider the 3-dimensional manifold M_n^1 determined by the points and interior points of a cube in 3-space when the points of opposite faces of the cube are matched with one another as they would be if one face were carried into its opposite by a translation. Again, we have

$$(P_1 - 1) = (P_2 - 1) = 3,$$

and there is no torsion. Each pair of opposite faces determines a non-bounding 2-circuit, each set of parallel edges, a non-bounding 1-circuit. In place of J_{22}, however, we obtain the matrix

$$(J'_{22}) \qquad \begin{array}{c|ccc} & S^1_2 & S^2_2 & S^3_2 \\ S^1_2 & 0 & S^3_1 & -S^2_1 \\ S^2_2 & -S^3_1 & 0 & S^1_1 \\ S^3_2 & S^2_1 & -S^1_1 & 0 \end{array}$$

This matrix is evidently not reducible to J_{22}, since no linear combination of the three rows with integer coefficients not all zero can be made to vanish.

A fuller report on the above matrices and on others obtained by studying the intersection of more than two systems of circuits will be made elsewhere. In particular I shall prove that the matrix J_{pq} may be replaced by a 3-dimensional array with integer elements.

The possibility of finding some such invariants as the ones obtained in this note was suggested a long time ago by Professor Veblen.

CONDITION THAT AN ELECTRON DESCRIBE A GEODESIC

BY ARTHUR BRAMLEY

DEPARTMENT OF PHYSICS, PRINCETON UNIVERSITY

Communicated January 17, 1924

Consider a Riemann space, whose linear element is written in the form

$$ds^2 = g_{ik}dx^i dx^k$$

The equations of motion of an electron[1] are

$$\mu g_{ij}\left(\frac{d^2x^j}{ds^2} + \Gamma^i_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}\right) = (S^k_i)_k \qquad (1)$$

where μ is the density and

$$\Gamma^i_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial g_{\alpha k}}{\partial x^\beta} + \frac{\partial g_{\beta k}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^k}\right)g^{ik}$$

If Maxwell's equations are

$$F_{\mu\nu} = \frac{\partial k_\mu}{\partial x^\nu} - \frac{\partial k_\nu}{\partial x^\mu}$$

and

$$F^{\mu\nu}_{\ \ \nu} = J^\mu$$

where

$$J^\mu = \rho\left(\frac{dx^1}{ds}, \frac{dx^2}{ds}, \frac{dx^3}{ds}, \frac{dx^4}{ds}\right)$$

and ρ is the charge density, then (2)

$$S_i^k = -F_{\alpha i} F^{\alpha k} + \frac{1}{4} F_{\alpha\beta} F_{\alpha\beta} \delta_i^k$$

The equation

$$S_i^k = T_i^k + \Delta_i^k$$

represents the most general law of conservation consistent with relativity mechanics, where T_i^k is the material energy tensor, and where Δ_i^k is a tensor which represents the effect of the non-Maxwellian forces operating on the electron.[2]

The set of equations

$$(Tij + \Delta ij - Phgij)\lambda^i h = 0$$

define an orthogonal ennuple, whose directions are by definition the principle directions of the tensor[3]

$$Tij + \Delta ij.$$

Then we have

$$gij\lambda^i h\lambda^j k = \delta hk$$

$$\begin{cases} \delta hk = 1 & k = h \\ 0 & k \pm h \end{cases}.$$

In order that these principle directions be completely indeterminate at every point, it is necessary and sufficient that the coefficient of the $\lambda^i h$ in the above equation be zero, i.e.,

$$Tij + \Delta ij = \rho gij \qquad (2)$$

where ρ is a scalar function.

Eisenhart[4] has shown that when the equation (1) of motion of an electron in general relativity are written in the form

$$\mu gij \left(\frac{d^2 x^j}{ds^i} + \Gamma^j_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} \right) = \phi i$$

then by a suitable change of the independent variable the vector ϕi can be identified with the gauge vector of Weyl's geometry. Thus we have

$$(Si^k)k = \phi i$$

If the principle directions are indeterminate for the tensor

$$\Delta ik + Tik,$$

then we have from (2)

$$(Ti^k + \Delta i^k)k = \frac{\partial \rho}{\partial x^i}$$

But

$$(Si^k)k = (Ti^k + \Delta i^k)k_2$$

hence

$$\phi i = \frac{\partial \rho}{\partial x^1}.$$

This equation shows that Weyl's gauge vector is necessarily a gradient; i.e. the forces acting on the electric particle are conservative since we can regard the ϕ's as the pondermotive forces of the field. The conservative nature of the forces gives a value clue to the conclusion that the system emits no radiation of energy.

This, however, has been shown by Eisenhart and Veblen[5] to be the condition that the Weyl's space be Riemannian, i.e. that the geodesics which the electron describes are defined by the equation

$$\frac{d^2x^i}{ds^2} + \Gamma^{\prime i}_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0$$

where

$$\Gamma^{\prime i}_{\alpha\beta} = \frac{1}{2} \lambda g^{ik} \left[\frac{\partial (\lambda g \alpha k)}{\partial x^\beta} + \frac{\partial (\lambda g \beta k)}{\partial x^\alpha} - \frac{\partial (\lambda g \alpha \beta)}{\partial x^k} \right]$$

and

$$-\rho = log\ \lambda.$$

The laws of mechanics are contained in the expression that the variation of the action is zero for any continuous deformation of the universe.

Suppose that the point x^i passes into the point $\bar{x}^i$ where

$$\bar{x}^i = x^i + \zeta^i(x_1 \ldots . x_n)$$

then $$\delta \int L dx' \ldots . dx'' = \delta \int L\, dx = 0$$

where L is such a function that

$$\delta L = \frac{1}{2} \mathbf{T}^{ik}\, \delta\, gik \qquad \mathbf{T} = T\sqrt{-g}$$

and the g is the determinant of the gik's.[6]

Due to the transformation of the coördinate system

$$-\delta\, gik = gir \frac{\partial \zeta^r}{\partial \pi^k} + gkr \frac{\partial \zeta^r}{\partial x^i} + \frac{\partial gik}{\partial \pi^r} \zeta^r$$

the gauge system remaining fixed.

Now

$$\delta \int L\, dx = \int \frac{\partial (L\zeta^i)}{\partial x^i} dx + \int \delta L\, dx$$

and

$$-\delta L = \mathbf{T}^k_i \frac{\partial \zeta^i}{\partial x^k} + \frac{1}{2} \mathbf{T}^{hs} \frac{\partial grs}{\partial xi} \zeta^i$$

If we define

$$\mathbf{V}^k_i = \mathbf{T}^k_i - \mathbf{L}.\delta^k_i$$

the equation may be written

$$-\delta \int L\, dx = \int \frac{\partial (\mathbf{V}^k_i \zeta^i)}{\partial x^k} dx + \int \left\{ -\frac{\partial \mathbf{T}^k_i}{\partial x^k} + \frac{1}{2} \frac{\partial grs}{\partial x^i} \mathbf{T}^{rs} \right\} \zeta^i dx$$

Since the variation ζ^i is entirely arbitrary, we must have

$$\frac{\partial (\mathbf{V}_i^k \zeta^i)}{\partial x^k} = \mathbf{V}_{,i}^k \frac{\partial \zeta^i}{\partial x^k} + \frac{\partial \mathbf{V}_i^k}{\partial x^k} \zeta^i = 0$$

and

$$\frac{\partial \mathbf{T}_i^k}{\partial x^k} - \frac{1}{2} \frac{\partial grs}{\partial x^i} \mathbf{T}^{rs} = T_{ik}^k = 0$$

The first equation leads to the conclusion that $V_i^k = 0$ whence

$$L. \delta_i^k = T_i^k$$

from which we have

$$L = \frac{1}{4} T.$$

The second of the above equations is the equation of mechanics for a continuous distribution of matter, while the first result shows that the principle directions of the tensor Tik are indeterminate.

If we impose the additional restriction that the principle directions of the tensor Δik are indeterminate, we have satisfied the conditions of the problem.

A solution satisfying this condition is evidently

$$\Delta ik = 0.$$

which corresponds to the ordinary law of conservation of energy. If we take

$$\Delta ik = \frac{R}{4} gik$$

where

$$R = Rijg^{ik}$$

Rij being the contracted curvature tensor, then the conditions are satisfied. The choice of Δij corresponds to the case considered by Einstein[7]. In both cases the space is homogeneous.[8]

Conversely, if the electron describes a geodesic in a Riemannian space and the laws of mechanics are valid, then the relation between electric and material energy tensors is necessarily of the form

$$Sik = Tik + \Delta ik \qquad (3)$$

where Δik is an arbitrary function whose principle directions are indeterminate.

In this case

$$\phi i = \frac{\partial \rho}{\partial x^i}$$

but $$\phi i = (Si^k)k = (Ti^k + \Delta i^k)k$$

thus $$(Ti^k + \Delta i^k - \rho.\delta i^k)k = 0$$

which proves the statement, since the principle directions of Ti^k are indeterminate.

Thus a necessary and sufficient condition that an electron describe a geodesic is that the principle directions of Δik are indeterminate.

[1] Eddington, *Mathematical Theory of Relativity*, p. 120.
[2] Eddington, *Ibid.*, p. 182.
[3] Ricci, *Atti R. Ist. Veneto*, **62**, 1230.
[4] *Proc. Nat. Acad. Sci. Washington*, **9**, 175 (1923).
[5] Eisenhart and Veblen, *Ibid.*, **8**, p. 19.
[6] Weyl, *Space, Time and Gravitation*, Section 28.
[7] Einstein, *Sitz. Pr. Ak. Wiss.*, April, 1919.
[8] Eisenhart, *Proc. Nat. Acad. Sci.*, **8**, p. 28.

LINKAGE OF DUTCH, ENGLISH, AND ANGORA IN RABBITS

BY W. E. CASTLE

BUSSEY INSTITUTION, HARVARD UNIVERSITY, BOSTON

Communicated January 17, 1924

In 1919[1] I presented evidence that in rabbits two patterns of white spotting, English and Dutch, are either allelomorphs or closely linked with each other. English pattern is dominant over self in crosses; Dutch may be called recessive, although Dutch cross-breds usually show the Dutch character feebly expressed. A cross between English and Dutch produces English marked rabbits, similar in appearance to homozygous English, although they are really English-Dutch heterozygotes. Such F_1 heterozygotes form two (and only two) kinds of gametes, so far as experiments have been carried. They transmit *either* English or Dutch, but no gametes are formed which transmit *neither* of these patterns as would be expected if the two are not allelomorphs. This conclusion was based on a study of 192 cases. Possibly a study of a larger number of cases would show the occurrence of cross-overs. For in rats we obtained no cross-overs between red-eyed yellow and albinism until some 400 or 500 cases had been studied. Then linkage was observed so nearly complete that only a fraction of one per cent of crossing-over has ever been found.

Subsequently it was found that English pattern is linked with angora coat. The cross-over percentage, as estimated from 1233 young produced by a back-cross between F_1 animals and double recessives, is 13.70 ± 0.96. For female F_1 parents alone, the case with which comparison will presently be made, the cross-over percentage is 11.94 ± 1.97.

If English and Dutch are linked with each other, and English is linked

with angora, Dutch also should be linked with angora; and if the linkage of English with Dutch is close, each should show about the same amount of linkage with angora as the other does. This has been found to be true.

In January 1921 a cross was made between self angora rabbits and Dutch short-haired rabbits. The F_1 animals were all short-haired and showed a low grade of Dutch, usually within grades 1–3[1]. The F_2 generation contained approximately the expected 25 per cent of angoras, and 25 per cent of high-grade Dutch (grades 12–16) but all of the latter were short-haired. This result indicated the existence of linkage, but gave no indication of its exact strength. To ascertain this the back-cross of F_1 with the double recessive was needed. But first the double recessive had to be produced, since none had appeared in F_2. By testing individual F_2 extracted homozygous (high-grade) Dutch against angoras, a few Dutch were found which were heterozygous for angora. In the production of such individuals, a cross-over gamete bearing both Dutch and angora had united with a non-cross-over gamete bearing Dutch and short hair.

Mating together these Dutch individuals heterozygous for angora, the desired double recessive combination was secured, homozygous both for Dutch and for angora. Two males of this constitution born in the spring of 1923 have now been back-crossed with F_1 females, giving a direct measure of the linkage by the departure observed from a 1:1:1:1 ratio. To date 68 young from these matings have been reared to an age when they can be classified as to hair-length. The Dutch pattern is of course recognizable at birth. These young include 37 high-grade (homozygous) Dutch, of which 36 are short-haired and one an angora. There are also 31 low-grade (heterozygous) Dutch, of which 25 are angora and 6 short-haired. The indicated cross-overs are thus 7 in 68 or 10.3 ± 4.09 per cent.

In testing linkage between English and angora, this same type of mating (F_1 female x double recessive male) has given 293 young with an indicated cross-over percentage of 11.94 ± 1.97, which, considering the size of the probable errors, is rather close to the result in the case of Dutch and angora.

Until larger numbers have been studied it will be impossible to say whether English and Dutch are allelomorphs or closely linked, but that they are both in a common linkage system with angora is fully established.

Races of Dutch rabbits vary greatly in the grade of development of the Dutch pattern. This I have explained[1] on the basis of multiple allelomorphism, but Punnett[2] and Pap[3] have interpreted it as due to multiple independent factors. A study of the linkage relations of different types of Dutch rabbits will afford a basis for deciding which of these conflicting views is correct.

[1] *Publ. Carnegie Inst. Wash.*, No. 288, 1919.
[2] Punnett, R. C., *J. Genet.*, **9**, March, 1920.
[3] Pap, E., *Zeit. ind. Abst. Vererb.*, **26**, 1921.

DISTINCTION BETWEEN PRIMARY AND SECONDARY CHROMOSOMAL MUTANTS IN DATURA

BY ALBERT F. BLAKESLEE

STATION FOR EXPERIMENTAL EVOLUTION, COLD SPRING HARBOR, N. Y.

Communicated January 30, 1924

Normal diploid ($2n$) plants of Datura Stramonium have 12 pairs of chromosomes in their somatic cells. It has been shown[1] that certain recurrent mutants in this species are due to the presence of a single extra chromosome in one of the 12 sets. In several of these ($2n + 1$) types, the evidence[2] is conclusive, we believe, that the extra chromosome is a specific one which brings about the peculiarities in the mutant by means of the unbalance which its factors produce over the normal $2n$ condition. Since there are in all only 12 chromosomal sets, one would expect consequently to find not over 12 mutants of the ($2n + 1$) type, each corresponding to the presence of a different extra chromosome. For some time, however, we have had more than 12 such mutants and at present there are over 20 which have been under cultivation for at least two generations.

The occurrence of so large a number of mutants which could be represented by the same chromosomal formula has naturally constituted our main problem with the Daturas for some time past. Evidence, however, has been accumulating from different sources which indicates that these 25-chromosome mutants are not all unrelated types; that, on the contrary, they may be arranged in not over 12 groups consisting of a single mutant or of a Primary mutant and one or more Secondary mutants. The present paper aims to give, in brief, evidence for the grouping of ($2n + 1$) mutants; evidence as to which are Primary and which Secondary mutants; and evidence as to the cause of the difference between the two.

I. Evidence for Groups of ($2n + 1$) *Mutants.*—That the ($2n + 1$) mutants arrange themselves into not over 12 groups is indicated by the facts in the following numbered paragraphs:

1. The presence of only 12 sets of chromosomes in the species is presumptive evidence for not over 12 groups to contain the 20 and more ($2n + 1$) types now known.

2. Similarities in external appearance alone would suffice in most cases in placing mutants in their respective groups. The similarities may affect habit, flower and leaf characters or apparently any other part of the plant. Capsules of members of three groups are figured in an earlier publication.[3]

3. A study of the internal anatomy made by Dr. Sinnott[4] has resulted in an arrangement of the mutants into groups which agree very closely with those made on the basis of other evidence.

4. In one group (Echinus-Mutilated), the pollen is dimorphic, half

the grains being supplied with starch and half being practically devoid of starch.

5. In one group (Poinsettia-Wiry) the two members give the same trisomic ratios[2] for the inheritance of a pair of Mendelian characters (cf. table 5).

6. The breeding behavior of the mutants indicates the grouping in that one member may throw another member of a group in its offspring (cf. II., 4 below & table 3).

7. The sizes of the extra chromosomes, which have been measured by Dr. Belling and which are discussed in the succeeding paper, offer additional evidence in regard to the grouping.

From the above evidence we have established 12 groups (if we include the mutant Spinach whose position is as yet doubtful), half of them with Secondaries and half in which no Secondaries have so far been found.

II. Evidence as to which Are Primary and which Are Secondary Mutants.—The grouping of mutants was established before it was possible to say with certainty which was to be considered the Primary and which the Secondary in a given group. The data, however, from special breeding tests between members of a single group as well as data collected for other purposes have afforded a series of tests for Primaries and Secondaries which leave little doubt in most cases as to the rank of the mutants within a group. These tests will be considered in numbered paragraphs.

TABLE I

PRIMARY AND SECONDARY ($2n + 1$) TYPES IN OFFSPRING FROM TRIPLOIDS

(Primaries are printed in capitals, secondaries in lower-case type)

	$3n \times$ SELF	$3n \times 2n$	TOTAL		$3n \times$ SELF	$3n \times 2n$	TOTAL
1. GLOBE	5	46	51	8. BUCKLING	9	48	57
2. POIN-				Strawberry	..	...	...
SETTIA	5	34	39	Maple			
Wiry	.	..	..	9. GLOSSY	..	...	...
3. COCKLE-		32	38	10. MICRO-	2	30	32
BUR	6	1	1	CARPIC	4	46	50
Wedge	.	33	37	11. ELONGATE	2	30	32
4. ILEX	4			Undulate	..	...	...
5. ECHINUS	3	15	18	12. SPINACH(?)	..	2	2
Mutilated	.	(2?)	(?)				
Nubbin(?)	.	..	..	Totals ($2n + 1$)	43	381	424
6. ROLLED	.	24	24	($2n + 1 + 1$)	11	101	112
Sugarloaf	.	..	..	$2n$	30	215	248
Polycarpic	.	..	..	$4n$	3	...	3
7. REDUCED	3	38	41	Grand Totals	87	697	784

1. Offspring from Triploids.—As has been shown,[5] diploids, simple ($2n + 1$) and double ($2n + 1 + 1$) mutants are to be expected in the offspring from triploid parents. Since $3n$ parents have an extra chromosome in each of the 12 sets, their offspring should show the 12 unmodified mutant types, each with a different extra chromosome, provided all such types were viable. A classification into their respective groups of 424 such mutant offspring is shown in table 1. Primaries are represented in capitals and Secondaries in lower case type as also in other tables throughout this paper. The position of Spinach from this and other tests is somewhat uncertain. It may possibly be a Secondary for which the Primary has

TABLE II

SPONTANEOUS OCCURRENCE OF PRIMARY AND SECONDARY ($2n + 1$) MUTANTS

(Primaries are printed in capitals, secondaries in lower-case type)

	FROM $2n$ PARENTS	FROM UNRELATED ($2n + 1$) PARENTS	TOTALS		FROM $2n$ PARENTS	FROM UNRELATED ($2n + 1$) PARENTS	TOTALS
1. GLOBE	41	107	148	8. BUCKLING	27	71	98
				Strawberry	1	1	2
2. POIN-				Maple	..	2	2
SETTIA	28	47	75				
Wiry	.	1	1	9. GLOSSY	8	11	19
3. COCKLE-				10. MICRO-			
BUR	7	17	24	CARPIC	64	100	164
Wedge	.	..	..				
				11. ELONGATE	..	2	2
4. ILEX	19	27	46	Undulate	..	1	1
5. ECHINUS	10	11	21	12. SPINACH(?)	6	4	10
Mutilated	2	4	6				
Nubbin(?)	1	..	1	Totals ($2n + 1$)	269	506	775
6. ROLLED	24	47	71	Related ($2n + 1$)			
Sugarloaf	3	9	12	types	....	22,123	22,123
Polycarpic	3	..	3	$2n$	32,523	70,281	102,804
7. REDUCED	25	44	69	Grand totals	32,792	92,910	125,027

not yet been distinguished from normals. The table seems to delimit with certainty, however, 11 of the possible 12 Primaries. It will be observed that there is only a single certain exception to the rule that triploids throw Primaries and not Secondaries.

2. Spontaneous Occurrence of Primaries and Secondaries.—In table 2 are shown the spontaneous occurrences of mutants among over 125,000 offspring from $2n$ and from unrelated ($2n + 1$) parents. By unrelated is meant not belonging to the same group as the new mutant. The tabulations for the $2n$ parents include the years 1918 to 1923; those for the

TABLE III

OFFSPRING OF PRIMARY AND SECONDARY ($2n + 1$) TYPES

(Primaries are printed in capitals, Secondaries in lower-case type)

	GL	PN	WY	CK	WD	IX	EC	MT	NB	EL	SG	PY	RD	BK	ST	MP	GS	MC	EL	UN	SP	TOTALS
POINSETTIA	14	8617	70	5		1				5	4		3	1				2	1		1	30933
Wiry		22	546							2												2998
COCKLEBUR	3	4		2034	1	1				4	1		2	4				3				8860
Wedge	6	6		36	472	1	2			3			1	3				5				2477
ECHINUS						1	182	0	0				1	1								718
Mutilated							37	376	0	1			1	3				4				2688
Nubbin(?)	2	2					13	0	122		1		3	6				6				400
ROLLED	22	16		3		4	3			2023	16	0	9	13	1	1	3	8			1	11140
Sugarloaf	6	1		2		2	1	1		155	1093	0	4	10			2	13			1	5230
Polycarpic										1	0	3					1					123
BUCKLING	4	7		1		4				6			4	1041	1	0	3	9				3101
Strawberry	3					3	1			2			2	36	287	0		4				947
Maple	1													4	0	422		1				1984
ELONGATE	1	1		1														3	66	0		1206
Undulate																			4	140		592

$(2n + 1)$ parents the years 1920 to 1923. Many different lines are represented in the table and the data for other reasons also are not completely comparable. They are probably sufficiently so, however, for our present purpose. It will be noted that in all cases the Primaries occur spontaneously much more frequently than their Secondaries which are relatively rare. Elongate occurs infrequently for a Primary, but its viability is known to be poor.

3. Percentage of Bad Pollen Grains.—In a preliminary study of pollen sterility undertaken with Mr. Cartledge it was shown that in all cases the Secondaries had a higher percentage of bad grains than their respective Primaries.

4. Breeding Behavior.—An inspection of table 3 will show that, whereas Primaries may occasionally throw their Secondaries, the Secondaries regularly throw their Primaries more frequently than they throw new mutants belonging to other groups. Thus, as shown in table 4, of the 31,000 offspring from Poinsettias about 28% were Poinsettia and about 0.25%

TABLE IV (CF. TABLE III)
OFFSPRING OF PRIMARY AND SECONDARY $(2n + 1)$ TYPES

	% PRIMARIES	% SECONDARIES	TOTAL
POINSETTIA	27.86	.23	30,933
Wiry	.73	18.31	2,998
COCKLEBUR	22.95	.01	8,860
Wedge	1.45	19.06	2,477
ECHINUS	25.35	0.0	718
Mutilated	1.38	13.99	2,688
Nubbin(?)	3.25	30.50	400
ROLLED	18.16	.14	11,140
Sugarloaf	2.96	20.90	5,230
Polycarpic	.81	2.44	123
BUCKLING	33.57	.03	3,101
Strawberry	3.80	30.31	947
Maple	.20	21.27	1,984
ELONGATE	5.50	0.0	1,206
Undulate	.68	23.65	592

were the Secondary Wiry. Conversely when Wirys were the parents, about 0.75% of the offspring were the Primary Poinsettia. In more than half the cases, Secondaries have arisen independently of their own group and have not yet been found among the offspring of their own Primaries. In all cases, however, the Secondaries have thrown their respective Primaries. The percentage of occurrences of Primaries in the offspring of Secondaries has varied from .20% in the case of Buckling from Maple to 3.8% in the case of Buckling from Strawberry.

5. Cytological Distinctions.—As is shown in the succeeding paper, Primaries and Secondaries, among those mutants for which adequate

cytological information has been secured, may be distinguished by differences in the configurations of their chromosomes.

III. Basis of Differences between Primaries and Secondaries.—The foregoing data point to the Secondary being a modified Primary. Rather extensive breeding experiments, which need not be given here, have proven untenable the provisional hypothesis that the modification might be brought about by a Mendelian factor. As a positive contribution, however, they seem to indicate that the modification is not carried by diploids nor by $1n$ gametes.

TABLE V

SEGREGATION OF MENDELIAN CHARACTERS IN $2n$ OFFSPRING IN TWO GROUPS OF PRIMARY AND SECONDARY $(2n + 1)$ TYPES

	POINSETTIA (P_2pAa)				Wiry ($PP'pAa$)			
	P	*p*	*A*	*a*	*P*	*p*	*A*	*a*
Selfed	1826	220	2155	738	98	14	116	38
Calculated	(1819)	(227)	(2170)	(723)	(100)	(12)	(116)	(39)
Ratio	8 :	1	3 :	1	8 :	1	3 :	1
Back-crossed	645	281	1103	940				
Calculated	(617)	(309)	(1022)	(1022)				
Ratio	2 :	1	1 :	1				
	COCKLEBUR (PpA_3a)				Wedge ($PpAA'a$)			
	P	*p*	*A*	*a*	*P*	*p*	*A*	*a*
Selfed	835	249	972	112	241	65	249	57
Calculated	(813)	(271)	(963)	(120)	(229)	(77)	(229)	(77)
Ratio	3 :	1	8 :	1	3 :	1	3 :	1
Back-crossed	271	238	1262	501	233	204	219	218
Calculated	(255)	(255)	(1175)	(588)	(219)	(219)	(219)	(219)
Ratio	1 :	1	2 :	1	1 :	1	1 :	1

More positive evidence as to the basis of the differences under consideration was secured from the breeding behavior of Wedge, which is a Secondary in the Cocklebur group. From table 5, in which for convenience only diploid offspring are shown, it will be seen that both Poinsettia and its Secondary Wiry give trisomic ratios for the color factors *P*, *p*, but give disomic ratios for spine factors *A*, *a*, indicating that both Poinsettia and Wiry have their extra chromosomes in the set carrying the factors *P*, *p* but not in the set with the factors *A*, *a*. Similarly the ratios for Cocklebur indicate that this Primary has its extra chromosome in the set carrying the factors *A*, *a*, but not in the set with factors *P*, *p*. Its Secondary Wedge, however, fails to give trisomic ratios for *A*, *a*. The ratios actually found resemble those in disomic rather than in trisomic inheritance and seem to indicate a deficiency in the extra chromosome of Wedge for the locus *A*, *a*, since the evidence strongly indicates that it is a Secondary of Cocklebur. If *A'* indicates the modified chromosome and *A* & *a* go to opposite poles at reduction division in a Wedge plant with the formula

$AA'a$, the gametes would be $A + a + AA' + aA'$. Such behavior would account for the ratios in table 5. If A' is deficient for the factor A, the gamete aA' would carry no factor for A; hence the disomic ratios between armed and *inermis* Wedges found but not shown in the table. If A & a occasionally should go to the same pole, the gametes would be A' (which would probably die) and Aa which would go to form a Primary Cocklebur occasionally thrown by Wedges.

The hypothesis of a deficiency in the extra chromosome of Secondaries has been strengthened by Dr. Belling's cytological findings. His hypothesis of reversed crossing-over, however, completes the picture by indicating a doubling of a part of the chromosome along with a deficiency of the remaining portion. The matter is considered in the paper which follows, but it should be pointed out that this new conception clears up many of the peculiarities in structure and breeding behavior of Primaries and Secondaries. Especially should be mentioned in this connection the complementary Secondaries in the Rolled and Buckling groups in which each Secondary shows certain characters manifested by the Primary but not shown by the other Secondary.

While we believe the problem of Primary and Secondary mutants is in way of being solved, we do not consider that all Secondaries are necessarily of the same type. Nubbin, for example, has been marked with a question in the tables because certain structural and breeding peculiarities seem to set it apart from the other Secondaries.

Summary.—I. The existence of not over 12 groups of $(2n + 1)$ mutants is indicated by the following more or less independent lines of evidence: 1. The occurrence of over 20 such mutants whereas there are only 12 sets of chromosomes in the species. 2. Similarities in external appearance. 3. Similarities in internal anatomy. 4. Pollen dimorphism in one group. 5. Trisomic ratios in one group. 6. Breeding behavior (cf. II, 4). 7. Chromosomal sizes (cf. next paper).

II. Evidence as to which are Primaries and which Secondaries is indicated by following evidence: 1. Primaries and not Secondaries occur in high proportions in offspring from triploids. 2. Primaries appear more frequently spontaneously. 3. Secondaries have a higher average proportion of bad grains in their pollen than their respective Primaries. 4. Secondaries occasionally come from Primaries, but Primaries regularly come from their Secondaries. 5. Differences between Primaries and Secondaries in configuration of their chromosomes (cf. next paper).

III. In seeking for a basis of the differences between Primaries and Secondaries, it has been shown, 1. that the difference is not due to a Mendelian factor, 2. that in one case the Secondary is modified apparently by deficiency of a portion of the extra chromosome, 3. that the cytological findings described in the succeeding paper give a chromosomal basis for

observed facts and are confirmed by the occurrence of complementary Secondaries.

[1] Blakeslee, Belling and Farnham, *Science, N. S.*, **52**, 388–90 (1920).
[2] Blakeslee and Farnham, *Amer. Nat.*, **57**, 481–95 (1923).
[3] Blakeslee, *Amer. Nat.*, **56**, 16–31 (1922).
[4] Sinnott and Blakeslee, These PROCEEDINGS, **8**, 17–19 (1922).
[5] Belling and Blakeslee, *Amer. Nat.*, **56**, 339–46(1922).

THE CONFIGURATIONS AND SIZES OF THE CHROMOSOMES IN THE TRIVALENTS OF 25-CHROMOSOME DATURAS

BY JOHN BELLING AND A. F. BLAKESLEE

CARNEGIE INSTITUTION OF WASHINGTON, DEPARTMENT OF GENETICS, COLD SPRING HARBOR, N. Y.

Communicated January 30, 1924

Among those progenies of triploids by normals[1] in which the chromosomes have been counted in all or nearly all of the sibs, eleven forms of 25-chromosome plants have occurred in not far from equal numbers. These same primary 25-chromosome forms occurred rarely in the progeny of diploid plants, doubtless arising wholly (or in part) from non-disjunction. Thus out of 1137 pollen-mother-cells of normal diploids (which comprise all plants whose chromosomes were counted, and may be reckoned a random sample) , there were found 8 cases of non-disjunction. In these cases, 11 and 13 chromosomes were present in the second metaphases, instead of 12 and 12. These would produce about 0.4 per cent of 13-chromosome pollen-grains. Non-disjunction occurs also, as might be expected, in the primaries themselves; so that other forms of 25-chromosome plants may spring from any one primary. In 521 pollen-mother-cells of the 11 primaries (and these were all the cells whose chromosomes were counted, and may be regarded as a random sample) there were 4 certain cases of non-disjunction in which two groups of 11 and 14 chromosomes were formed. Since non-disjunction (as distinguished from random distribution of the odd chromosome) changing 12 and 13 into 13 and 12 cannot be ascertained by mere counting, and should be equally frequent with the 11 and 14 distribution; there were probably nearly as many cases of new groups of 13 as the number of groups of 14 seen. Hence, in the pollen of any primary, there might be about 0.4 per cent of 13-chromosome grains having an extra chromosome different from the normal extra chromosome of that primary. Doubtless many of these 13-chromosome grains do not function. But if the same amount of non-disjunction occurs in the egg-cells, we might expect

in the progeny of both diploids and primary 25-chromosome plants, a number of new primary 25- chromosome forms; whose maxmimum amounts of about 0.7 per cent, or 0.35 per cent (according as the 13-chromosome pollen does, or does not, function) would probably not be reached, because of differences of viability of the resulting zygotes.

Out of 109 trivalents of 10 of the primary 25-chromosome plants (which trivalents may be regarded as forming a random sample), 50 had the shape

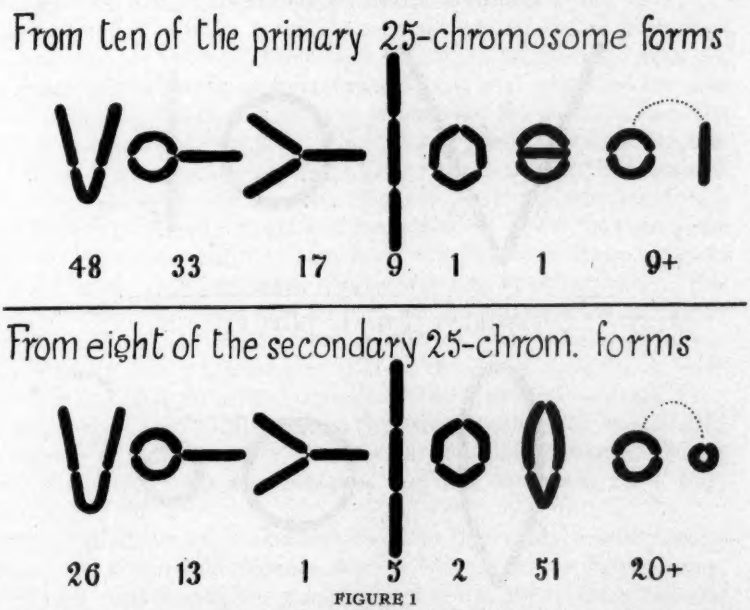


FIGURE 1

Upper row.—Diagram of a random sample of 109 trivalents of ten of the primary 25-chromosome forms (found commonly in the progeny of triploids). The results of 9 cases in which one of the three homologous chromosomes was separate, are included. The commonest form is the *V*. The rare triple arc occurs in only 1 instance. The presence of one hexagon does not agree with the working hypothesis.

Lower row.—Diagram of a random sample of 98 trivalents of 8 of the secondary 25-chromosome forms. The commonest form is the *V*-ring. In 20 cases the separated odd chromosome was itself bent into a small ring.

of a ring-and-rod, or of a *Y* (fig. 1); 48 were *V*'s, the middle chromosome being bent, and the two others nearly straight; 9 formed a chain of three; while the other forms were rare. If we assume that similar ends of homologous chromosomes come together in synapsis, and if we designate the two ends of the normal chromosomes by *a* and *z*; in the ring-and-rod, two *a*'s and three *z*'s are combined, and one *a* is free. The *Y* is seemingly less stable, since all three *a*'s are uncombined. The *V* (fig. 2) and the chain of three

have presumably two z's and two a's combined; and since only one a and one z are left free, the combination seems stable. The triple arc has the 3 a's and the 3 z's combined, but it is rare. Another rare form, the hexagon, cannot be suited to this scheme; though the other configurations found in the triploid and tetraploid[2] agree with this method of considering them. (The V is found to be common in the triploid, where it has been confused

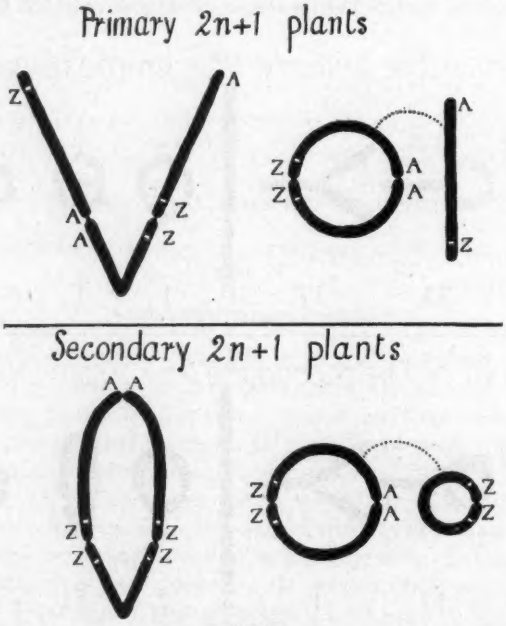


FIGURE 2

Upper half.—Diagram showing hypothetical situation of homologous ends of the chromosomes in the ordinary V trivalents of the primaries. If these two straight chromosomes were parallel in the thin-thread stage, ordinary crossing-over could not occur. On the right is an attempted explanation of the straightness of the detached chromosome.

Lower half.—Diagram of the V-ring of the secondaries on the hypothesis that homologous ends of chromosomes are in contact. Sometimes the 4 Z-ends meet to form a figure-of-eight. On the right is a hypothetical explanation of the small circle made by the detached chromosome.

with a Y.) It is obvious that the Y, and the chain of three, may have come from the opening out of the ring of the ring-and-rod, at different ends. In this configuration (ring-and-rod), crossing-over may have occurred normally at synapsis between the two chromosomes forming the ring. Sometimes the V trivalent, when traced back as far as possible towards the thin thread stage, still remains a V, the two end chromosomes being more or

less parallel. Normal crossing-over could not occur here, since the two parallel chromosomes are presumably in reverse positions, the *a* end of each being opposite the *z* end of the other. (fig. 2.). (If crossing-over, i.e., segmental interchange, occurs nevertheless, the only point where the loci are identical is at the center of the two chromosomes, or of their overlapping parts. In this case two chromosomes would result, one with an *a* at each end, and the other with a *z* at each end. The same result might occur in other configurations, by a somewhat different process.)

Besides the 11 primary forms, at least 8 secondary forms of 25-chromosome plants are known. These have originated infrequently from the corresponding primaries or from triploids; and also from diploids and other primaries. In these 8 forms of secondaries, 165 pollen-mother-cells gave no case of non-disjunction. This shows, at least, that non-disjunction is not much more common in secondaries than in diploids or primaries. These secondaries showed one main difference in the configuration of their trivalents as compared with those of the primaries. Six of the eight forms (and of the other two only 1 to 2 trivalents were examined) showed 51 cases of a *V*-ring (fig. 1), a form which has not been seen in any primary. This ring of three chromosomes with one bent into a *V* cannot be brought under the foregoing scheme, unless we assume that the bent chromosome (or one of the others) is a *zz* chromosome or an *aa* chromosome (fig. 2); in each case formed out of the similar segments of two homologous chromosomes, as mentioned above. Out of the 98 trivalents examined, 51 were *V*-rings as already defined, 26 were *V*'s, 13 had the ring-and-rod configuration, 5 were chains of three, 2 were hexagons, and only one was of the *Y* form (fig. 2).

In both primaries and secondaries, one of the three characteristic chromosomes sometimes remains separate, often outside the first metaphase group. In all cases examined in the primaries, and also in the triploids, this odd chromosome was straight or but slightly curved. In 20 cases from three different secondaries (which were all that were seen) the odd chromosome formed a small ring. This might be expected on the supposition that the two ends of this chromosome were similar (homologous). In these cases, the other chromosomes of the trivalent usually combined to form a ring of two.

From each of four primaries and two secondaries, 12 trivalents were drawn with the camera under the same magnification (over 2000). These trivalents were taken in sequence, and form a random sample. Measurements of the drawings were made under a binocular magnification of 3.5. (Length and average breadth were measured.)

The chromosomes of Datura Stramonium form six or seven classes when arranged according to size, namely: one largest; four large, one of which seems larger than the other three; three large medium; two small medium; one small; and one smallest.

The primary with the largest extra chromosome (Rolled) gave as the mean product of the lengths and breadths of 36 chromosomes in the trivalents; 54.9 ± 0.9. One of the two secondaries with the largest extra chromosome (Sugarloaf) gave 52.9 ± 0.9. The difference is 2 ± 1.3. Since this difference is less than twice its probable error, the chromosomes of this secondary are not statistically different in size from those of the corresponding primary.

Another primary (Echinus) doubtless with the large medium size of extra chromosome, gave as the product of length by breadth for 36 chromosomes; 32.1 ± 0.3. The corresponding secondary (Mutilated) gave 31.9 ± 0.3. The difference was negligible; 0.2 ± 0.4. Another primary (Microcarpic) of the same size class gave 30.9 ± 0.3.

The primary (Globe) whose trivalent belonged to the small size class gave, for 36 chromosomes, 20.9 ± 0.4. Its secondary has not been measured.

Hence, so far as these measurements go, there is no demonstrable difference in chromosome size between a primary and its secondary.

Summary.—(1) The primary 25-chromosome forms arise from the normal chromosome distribution in triploids, pollinated by diploids or selfed; or from non-disjunction in diploids, or in other primaries. This non-disjunction results in about one-third per cent of 13-chromosome pollen (and possibly egg-cells).

(2) The primary forms have trivalents in which the open *V* predominates. The odd chromosome when detached is mostly straight.

(3) The secondary 25-chromosome plants come occasionally from corresponding primaries (including triploids), and also from diploids and other primaries. Their method of formation possibly consists in a change in the chromosome concerned, so that its two ends are homologous.

(4) Among the trivalents of the secondaries, the closed *V*-ring is usually predominant. The odd chromosome detached from the trivalent is often rolled into a small ring.

(5) The working hypothesis of an occasional reversed crossing-over, or interchange of non-homologous segments between two chromosomes of a trivalent, is being used until a better hypothesis is found.

[1] Belling, J. and A. F. Blakeslee, 1922, "The Assortment of Chromosomes in Triploid Daturas," *Amer. Nat.*, **56**, 339–346.

[2] Belling, J. and A. F. Blakeslee, 1923, "The Reduction Division in Haploid, Diploid, Triploid and Tetraploid Daturas," These PROCEEDINGS, **9**, 106–111.